The Solid State

1. The edge length of unit cell of a metal having molecular weig t 75 g/ ol is 5Å which crystallizes in cubic lattice. If the density is 2 g/cc and the radius of metal atom is 43.3 x pm. ($N_A = 6 \times 10^{23}$). Find the value of x.

2. In face centred cubic (fcc) crystal lattice, edge length is 400 pm. The diameter of greatest sphere is 29.29 d pm which can be fit into the interstitial void without distortion of lattice. Find the value of d.

3. AB; crystallizes in a body centred cubic lattice with edge length 'a' equal to 387 pm. What is the distance between two oppositely charged ions in the lattice?

4. If calcium crystallizes in bcc arrangement and the radius of Ca atom is 96 pm, and the volume of unit cell of Ca is $x \times 10^{-30}$ m^3. Find the value of x.

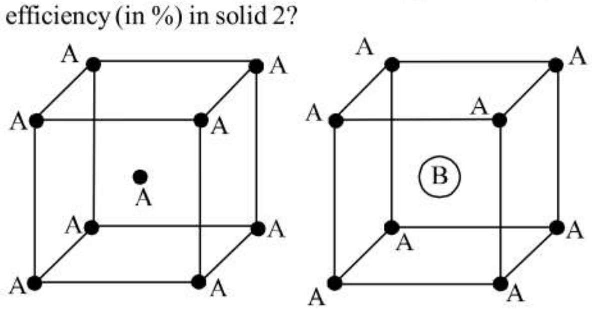
5. The radii of Na^+ and Cl^- ions are 100 pm and 200 pm respectively. Calculate the edge length of NaCl unit cell.

6. CsBr has *bcc* structure with edge length 4.3Å. What is the shortest interionic distance (in Å) between Cs^+ and Br^-.

7. A metallic crystal crystallizes into a lattice containing a sequence of layers AB AB AB......Any packing of spheres leaves out voids in the lattice. What percentage of volume of this lattice is empty space?

8. A molecule A_2B (Mwt. = 166.4) occupies triclinic lattice with a = 5 Å, b = 8 Å, and c = 4 Å. If the density of AB_2 is 5.2 g cm^{-3}, find the number of molecules present in one unit cell.

9. Consider the bcc unit cells of the solids 1 and 2 with the position of atoms are shown below. The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficiency (in %) in solid 2?



Solid 1 Solid 2

10. Find the sum of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure.

11. Calculate the radius (in pm) of the largest sphere which fits properly at the centre of the edge of a body centred cubic unit cell. (Given edge length is 100 pm)

12. CuCl has face centred cubic structure. Its density is 3.4 g cm^{-3}. What is the length of unit cell in Å?

13. Molybdenum forms body-centred cubic crystals whose density is 10.3 g cm^{-3}. Calculate the edge length of the unit cell. The molar mass of Mo is 95.94 g mol^{-1}.

14. The density of mercury is 13.6 g/mL. Calculate approximately the diameter in Å of an atom of mercury assuming that each atom is occupying a cube of edge length equal to the diameter of the mercury atom.

15. In the fluorite structure if the radius ratio is $\left(\sqrt{\frac{3}{2}} - 1\right)$, how many total ions does each cation touch?

11. **(6.7)** For *bcc*

$$\Rightarrow R = \frac{\sqrt{3}\,a}{4}$$

$\therefore$ Empty space at edge $= a - 2R = a - \frac{\sqrt{3}\,a}{2}$

$$\therefore \quad r_{sphere} = \frac{a - \frac{\sqrt{3}}{2}a}{2} = \left(\frac{2-\sqrt{3}}{4}\right)a$$

= diameter of sphere.

$= 0.067\,a$ (Given a = 100 pm)

$= 0.067 \times 100 = 6.7$ pm

12. **(5.78)** Molecular mass of CuCl = 99

$n = 4$ for face centred cubic cell

$$\text{Density} = = \frac{n \times \text{mol.wt.}}{V \times \text{Av.no.}} = \frac{4 \times 99}{a^3 \times 6.023 \times 10^{23}}$$

$$\text{or } 3.4 = \frac{4 \times 99}{a^3 \times 6.023 \times 10^{23}}$$

$a = 5.783 \times 10^{-8}$ cm $= 5.783$ Å

13. **(314)** Given, *bcc*, i.e., Z = 2

Crystal density = 10.3 g/cm^3

M = 95.94 g/mol

Apply the formula,

$$\text{Crystal density} = \frac{Z \times M}{V \times N_A}$$

$$10.3 = \frac{2 \times 95.94}{V \times 6.02 \times 10^{23}}$$

$V = 3.0945 \times 10^{-23}$ cm^3

$= 3.0945 \times 10^{7}$ pm

$\because \; V = a^3$

$\therefore \; a = (V)^{1/3}$

$a = 313.9$ pm

14. **(2.91)** $N_A = 6.023 \times 10^{23}$

Atomic mass of mercury = 200

$\therefore$ Number of atoms present in 200 g of Hg $= 6.023 \times 10^{23}$

So Number of atoms of Hg present in 1g of Hg

$$= \frac{6.023 \times 10^{23}}{200} = 3.0115 \times 10^{21}$$

Density of mercury = 13.6 g/cc

$\therefore$ Volume of 1 atom of mercury (Hg)

$$= \frac{1}{3.0115 \times 10^{21} \times 13.6}\text{cc} = 2.44 \times 10^{-23}\text{ cc.}$$

Since each mercury atom occupies a cube of edge length equal to its diameter, therefore

Diameter of an atom of mercury

$= (2.44 \times 10^{-23})^{1/3}$ cm.

$= 2.905 \times 10^{-8}$ cm $= 2.91$ Å

15. **(20)** $\frac{r_+}{r_-} = \sqrt{\frac{3}{2}} - 1 = 0.225$

Hence, it is the limiting case where cation in the void of fcc structure is not distored.

Note: A fluorite-type structure has ccp arrangement in which cation (Ca^{2+} ion) forms fcc arrangement with each Ca^{2+} ion surrounded by 8 anions (F^- ions) and each anion (F^- ion) surrounded by 4 cations (Ca^{2+} ion).

So, number of cations surrounding the particular cation = 12. But at the same time 8 anions (present in TVs) touch the particular cation.

SOLUTIONS

1. **(5)** $d = \dfrac{n \times M}{N_A \times a^3}$ or $n = \dfrac{d \times N_A \times a^3}{M}$

$\Rightarrow n = \dfrac{2 \times 6 \times 10^{23}\ (5 \times 10^{-8})^3}{75} = 2$

Therefore Metal crystallizes in *bcc* structure and

for a *bcc* lattice $\sqrt{3}a = 4r$

$r = \dfrac{\sqrt{3}}{4}a = \dfrac{\sqrt{3} \times 5}{4} = 2.165Å = 216.5$ pm

$x = \dfrac{216.5}{43.3}$

$x = 5$

. **(4)** For an octhedral void $a = 2(r + R)$

In fcc lattice the largest void present is octahedral void. If the radius of void sphere is R and of lattice sphere is r,

Then, $r = \dfrac{\sqrt{2} \times 400}{4} = 141.42$ pm   ($a = 400$ pm)

Applying condition for octahedral void,

$2(r + R) = a$

$\therefore 2R = a - 2r = 400 - 2 \times 141.42$

$\therefore$ Diameter of greatest sphere $= 117.16$ pm

$d = \dfrac{117.16}{29.29} = 4$

3. **(335)** For *bcc* lattice body diagonal $= a\sqrt{3}$.

The distance between the two oppositely charged ions

$= \dfrac{a}{2}\sqrt{3} = \dfrac{387 \times 1.732}{2} = 335$ pm

4. **(10.9)** For bcc lattice, $\sqrt{3}a = 4R \Rightarrow a = \dfrac{4 \times 96}{\sqrt{3}}$ pm

$= 221.7$ pm

(where R is the radius of Ca atom)

Volume of unit cell $= a^3 = (211.7 \times 10^{-12})^3\ m^3$

$= 10.9 \times 10^{-30}\ m^3$

Thus, $10.9 \times 10^{-30} = x \times 10^{-30}$

$x = 10.9$

5. **(600)** In a *fcc* lattice, the distance between the cation and anion is equal to the sum of their radii, which is equal to half of the edge length of unit cell,

i.e. $r^+ + r^- = \dfrac{a}{2}$ (where a = edge length)

$r^+ = 100$ pm, $r^- = 200$ pm

Edge length $= 2r^+ + 2r^- = (2 \times 100 + 2 \times 200)$ pm

$= (200 + 400)$ pm $= 600$ pm.

6. **(3.72)** For *bcc* structure, atomic radius, $r = \dfrac{\sqrt{3}}{4}a$

$= \dfrac{\sqrt{3}}{4} \times 4.3 = 1.86Å$

Since, r = half the distance between two nearest neighbouring atoms.

$\therefore$ Shortest interionic distance $= 2 \times 1.86 = 3.72Å$

7. **(26)** In AB AB packing spheres occupy 74%. 26% is empty.

8. **(3)** Volume of unit cell $= a \times b \times c$

$= 5 \times 10^{-8} \times 8 \times 10^{-8} \times 4 \times 10^{-8}$

$= 1.6 \times 10^{-22}\ cm^3$

Mass of unit cell $= 1.6 \times 10^{-22} \times 5.2$

$= 8.32 \times 10^{-22}$ g

Number of molecules in one unit cell

$= \dfrac{8.32 \times 10^{-22}\ g}{166.4\ g\ mol^{-1}} = 3$

9. **(90)** Volume occupied by atoms in solid 2

$= \dfrac{4}{3}\pi r^3 + \dfrac{4}{3}\pi(2r)^3 = 12\pi r^3$

Relationship between body diagonal and radius of atom (r),

$6r = \sqrt{3}a$

$a = \dfrac{6r}{\sqrt{3}}$

Packing efficiency $= \dfrac{\text{vol. of particles}}{\text{vol. of unit cells}}$

Packing efficiency $= \dfrac{12\pi r^3}{\left(\dfrac{6r}{\sqrt{3}}\right)^3} \times 100 = 90\%$

10. **(7)**

Unit cell	**No. of atoms**
Simple cubic	$\frac{1}{8} \times 8 = 1$
bcc	$\frac{1}{8} \times 8 + 1 \times 1 = 2$
fcc	$\frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$

Thus, the sum is 7.